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LAMINAR COMPRESSIBLE MIXING BEHIND FINITE BASES

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November 1963

P-2828

LAMINAR COMPRESSIBLE MIXING BEHIND FINITE BASES

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In Ref. 1 Chapman considered the problem of laminar mixing at constant pressure for a fluid with Prandtl number one, and a viscosity power law of the form $\mu \sim T^{\omega}$. Because of the recent interest in the fluid-mechanic description of hypersonic wakes, mixing problems are being again investigated intensely both experimentally and theoretically. As an example, Ref. 2 extends Ref. 1 by assuming a Blasius starting velocity profile rather than a uniform one. (A uniform profile was assumed by Chapman as a necessity imposed for the conservation of similarity).

It will be shown in this brief note that the velocity along the dividing streamline can be determined in a simple manner by approximating the integral solution of Ref. 1. The effect of finite base radius (or Reynolds number) is also investigated using this approximate solution.

In terms of the stream function Ψ, the differential equation of motion is:

$$\frac{\zeta}{2} \frac{du}{d\zeta} + \frac{d}{d\zeta}\left(g \frac{du}{d\zeta}\right) = 0 \qquad (1)$$

where

$$\zeta = \Psi/\sqrt{U\nu sC}, \qquad g(\zeta) = uT^{\omega-1}$$

$$T = T_d - \frac{\gamma-1}{2} M^2 u^2 + (T_o - T_d)u.$$

T and u are nondimensional with respect to the free-stream, the subscripts d and o refer to the "dead water" region and stagnation point,

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and M is the free-stream Mach number. Chapman's boundary conditions are:

$$\text{At } \zeta = \infty: \ u = 1, \qquad \zeta = -\infty: \ u = 0 \tag{2}$$

The velocity at the dividing streamline, u_D, is given after a formal solution of Eq. (1) as follows:

$$u_D = \frac{\int_{-\infty}^{0} F d\zeta}{\int_{-\infty}^{0} F d\zeta + \int_{0}^{+\infty} F d\zeta} \tag{3}$$

where

$$F(\zeta) = \frac{\exp\left\{-\int_{0}^{\zeta} \frac{\zeta}{2g} d\zeta\right\}}{g} \tag{4}$$

We make the observation that in the expression for the integrand F, the contribution of the exponential part in the numerator is stronger than the denominator for increasing g (or ζ). Hence in the intervals I: $-\infty < \zeta \leq 0$ and II: $0 \leq \zeta < +\infty$ most of the contribution comes about from the values of g corresponding to the highest ζ inside the interval. For a first iteration let us therefore assume for u the following step function: Inside I, $u = u_D$; Inside II, $u = 1$. Simple integration of Eq. (3) yields

$$u_D = \frac{1}{1 + \sqrt{u_D T_D^{\omega-1}}} \tag{5}$$

Assuming that $T_d = T_o$ and $\omega = 0.75$ the calculations show that for M = 0, 1, 2, 3, 4, 5, 7, 10, 15, 20, the corresponding values of u_D are: 0.570, 0.573, 0.581, 0.590, 0.598, 0.605, 0.619, 0.634, 0.652, 0.664. For M = 0 and 5 Chapman[3] gives, through an exact numerical solution, u_D = 0.587 and 0.597. Comparison shows that our closed form approximation is in error of less than -3% and +1.5% correspondingly.

Experiments show[4] that u_D is a function of Reynolds number. The analysis of Ref. 2, in which the influence of an initial finite boundary layer thickness was studied, through the parameter $s^* \sim s/s_b$, yields results which are independent of the base radius r_o. (This occurs because as it can be seen from Fig. 1, $r_o = s_b \sin \alpha = s_n \sin \beta$). One might conjecture that one way to introduce the Reynolds number would be to assume that $u = 0$ not at $\zeta = -\infty$ but at $\zeta = -\zeta_n$ where ζ_n is finite and positive. This assumption implies a finite radius in the direction perpendicular to the main flow. Following the same method used for the derivation of Eq. (5) we find:

$$u_D = \frac{\text{Erf.}\ (\chi_n)}{\text{Erf.}\ (\chi_n) + \sqrt{g_D}} \tag{6}$$

where $\chi_n = \zeta_n/2\sqrt{g_D}$ and Erf. denotes the error function. Figure 2 shows the function $u_D(M, \zeta_n)$ for three different Mach numbers.

These results are best interpreted in the physical plane s,y. For simplicity assume $M = 0$ so that[1]

$$y\sqrt{\frac{u}{\nu s}} = \int_0^{\zeta} \frac{d\zeta}{u} \tag{7}$$

Let ζ_n correspond to y_n and s_n where the subscript n indicates the position of the "neck" of thickness h as shown in Fig. 1. To calculate the integral in Eq. (7) we approximate $u(\zeta)$ in the interval $0 \leq \zeta \leq \zeta_n$ by dropping the first term in Eq. (1). A simple integration yields:

$$u = u_D\sqrt{1 - \frac{\zeta}{\zeta_n}} \tag{8}$$

Introducing the above into Eq. (7) we have:

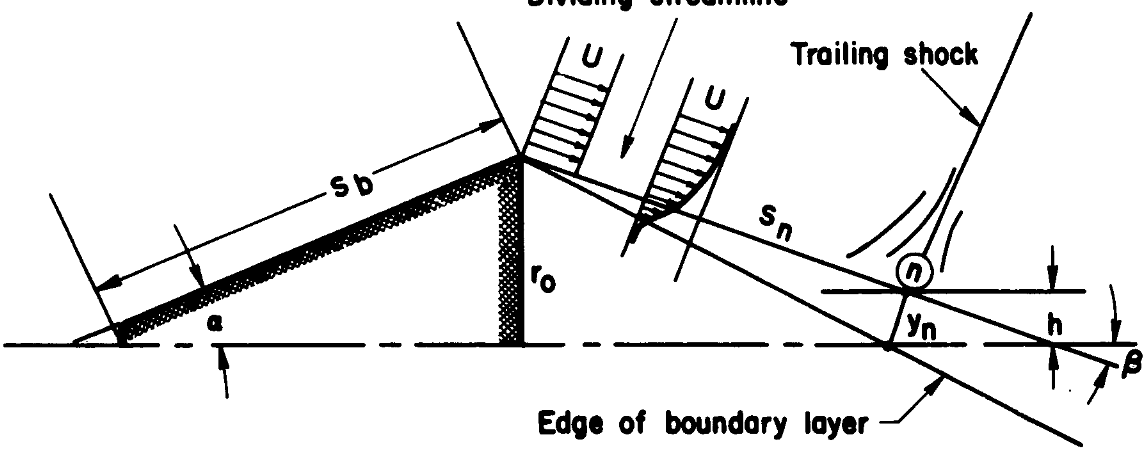

Fig. 1 — Schematic diagram of base flow

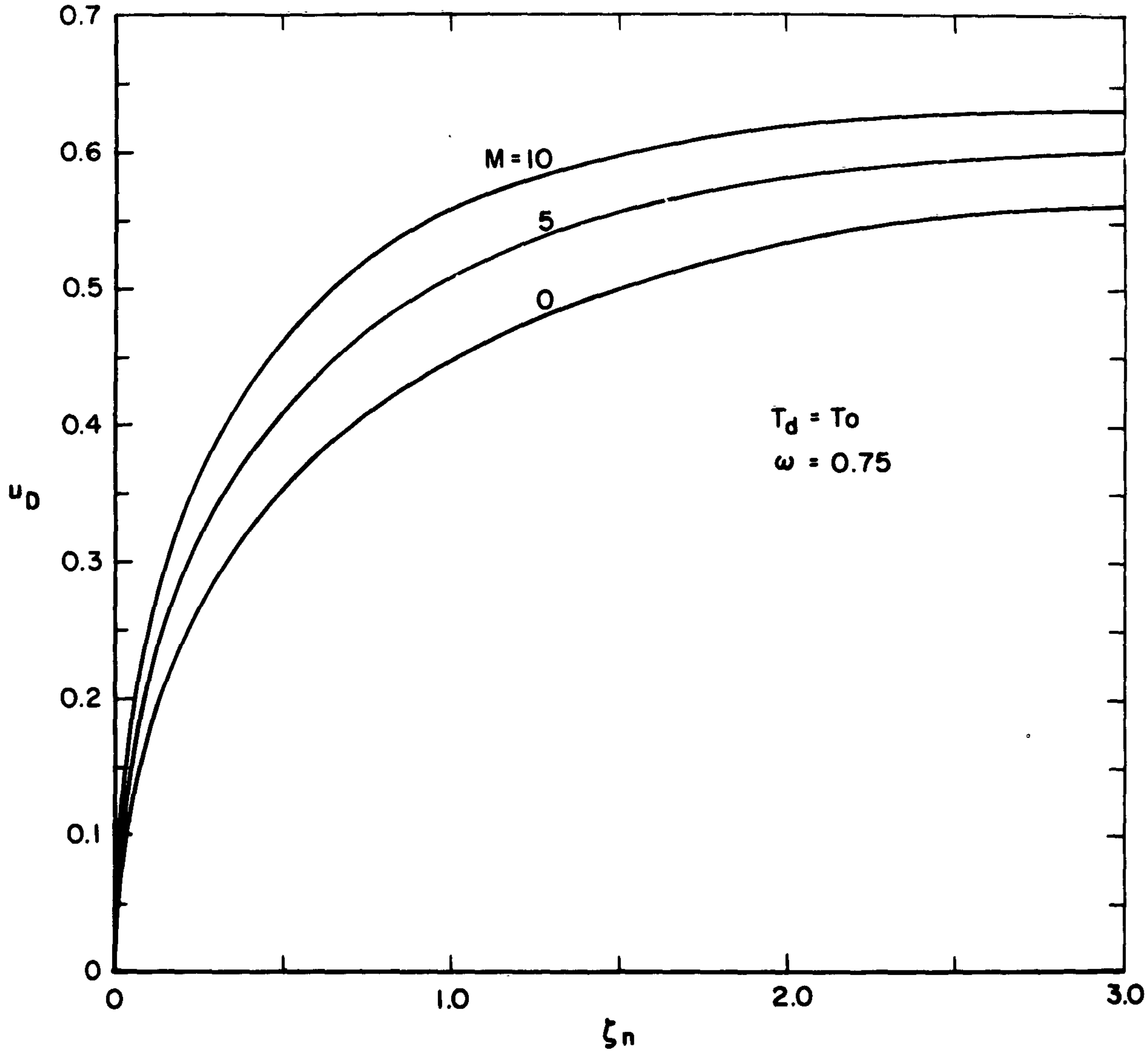

Fig. 2 — The velocity at the dividing streamline for different boundary conditions inside the base region

$$y_n \sqrt{\frac{u}{\nu s_n}} = 2 \frac{\zeta_n}{u_D} \tag{9}$$

Assuming that all of the mass contained in the boundary layer over the body goes through the neck, we set

$$h/r_o = 40 / \sqrt{\frac{Ur_o}{\nu}} \tag{10}$$

The factor 40 is fixed by the experiments of Ref. 3. Setting the position of the neck λ radii back of the base, Eq. (9) yields, after some trigonometry:

$$\zeta_n = 20\, u_D / \sqrt{\lambda \cos \beta} \tag{11}$$

From this equation it appears that the value of u_D is again directly independent of the Reynolds number. Since the angle β is of the order of 10^o, so that $\cos \beta \approx 1$, and since from experiments, λ is of order one, the assumption $\zeta_n \to \infty$ is still reasonable. The proof of this last statement lies in the fact that Eq. (11) yields a value of $\zeta_n = 12$ when $u_D \approx 0.6$, $\lambda \approx 1$ and $\cos \beta \sim 1$; then, Fig. 2 indicates that our choice for u_D is consistent with the fact that at $\zeta_n = 12$ the asymptotic value for u_D has been reached. In fact it is reached roughly when $\zeta_n > 3.0$. One needs a neck length of the order of 10 radii in order to make a correction in u_D for finite base radius. However, such values are not observed experimentally.

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